Инженерный вестник Дона, №11 (2024)
ivdon.ru/ru/magazine/archive/n11y2024/9639

Multifractal Analysis for Young's Modulus Estimation in Composite Pipes

Kh. M. Vafaeva

Peter the Great St.Petersburg Polytechnic University, Saint Petersburg, Russian Federation

Abstract: The research aimed to investigate the structure of glass-basalt composite pipes and explore the relationship between their Renyi statistical dimensions and physicomechanical properties. Physical experiments were conducted to measure and analyze the elasticity of glass-basalt composite pipes. The experiments included testing the modulus of elasticity and other mechanical properties. Fractal analysis was applied at the microstructural level to assess the influence of the fiber matrix structure on the physicomechanical behavior of the pipes. The study explored the possibility of modeling the microstructure of glass-basalt composite pipes using 3D fractal analysis. A correlation was established between the spectrum of multifractal dimensions (D_{-200}, D_0, D_1, D_2, D_{200}), the heterogeneity of the fiber matrix $f(\alpha)$, and the elasticity properties (Young's modulus). For the obtained fractal models predicting Young's modulus, the correlation coefficients (R^2) were 0.95 for D_0, 0.92 for D_1, 0.90 for D_2, 0.82 for D_{-200}, and 0.68 for $f(\alpha)$. These results can be applied for rapid estimation of Young's modulus using optical microscopy and photomicrographs of the microstructure.

Keywords: fractal modeling, microstructure, physicomechanical properties, material development, glass-basalt fiber, polymer pipes, forecasting, interphase boundaries, mechanical properties, fractal dimension, heterogeneity.

1 Introduction

The properties of a material depend on its structure, phase, mineralogical, and chemical composition, which vary according to its nature [1]. The structure of real materials is complex, as it typically consists of both homogeneous and heterogeneous regions [2]. The physicomechanical properties of materials are significantly influenced by interatomic bonds [3], physical characteristics (such as density, melting temperature, etc.) [4], structural defects [5], and other structural features [6]. The composition and manufacturing technology of glass-basalt composite pipes [7] determine the heterogeneous morphology of their surface [8], while the modulus of elasticity (Young's modulus) [9] is one of the most important characteristics, as specified by regulatory documents, which describes their suitability for operation under various conditions.

It is known that Young's modulus (E) is a physical quantity that describes a material's ability to resist stretching or compression under elastic deformation [10]:

$$E = \frac{F / S}{\Delta l / l} = \frac{F \cdot l}{S \cdot \Delta l}, \tag{1}$$

where F is the normal component of the force, S is the surface area over which the force is distributed, and Δl is the change in length of the rod (l) due to elastic deformation. Young's modulus (1) is related to key parameters such as the material's density, the total elementary work involved in forming the fracture surface [11], and the Poisson's ratio [12].

When dealing with a homogeneous (isotropic) body, the modulus E is directly proportional to the shear modulus G (2) and the bulk modulus K (3) through the following relationships:

$$G = \frac{E}{2 \cdot (1 + \nu)}, \quad K = \frac{E}{3 \cdot (1 - 2\nu)}. \tag{2}$$

Glass-basalt composite materials exhibit exceptional compressive and tensile strength, making them an ideal choice for Arctic conditions, which are characterized by significant mechanical loads. With a compressive strength of 100 MPa and a tensile strength of 8 MPa, glass-basalt composite materials surpass concrete and steel in these properties. Concrete demonstrates a strength range of 50-75 MPa, while steel shows strengths ranging from 250 to 500 MPa. However, both concrete and steel are susceptible to corrosion and wear in harsh Arctic conditions. Glass-basalt composite materials [13] have a hardness rating of 8 on the Mohs scale, indicating a high level of hardness [14]. Concrete and steel, with hardness values ranging from 3-4 and 5-8 respectively, show lower resistance to wear compared to glass-basalt composite materials [15]. Therefore, glass-basalt composites are a promising material for the future [16].

Currently, there are no unified models for evaluating the elasticity properties of glass-basalt composite materials, including tube products, based on the influence of structural characteristics. One of the reasons for this is the heterogeneity of their structure.

In this study, the fractal geometry approach [17], specifically the multifractal formalism, was employed to evaluate the Young's modulus of glass-basalt composite tubes, allowing for the assessment of heterogeneous and geometrically complex regions of the structure [18].

Fractal theory is based on intermediate asymptotics, which is characteristic of the structures of many objects, including the structure of various materials. The fractal theory has been successfully applied to model the structure [19] and properties of concrete [20], as well as for microstructural and micro-analytical analysis of concrete exposed to sulfate attack [21] and metallic materials [22]. For instance, in [23], a new finite element model for evaluating the Young's modulus of metallic foams, produced using powder metallurgy and a phase-holder space, is discussed. The model was based on volume reduction and reproducing the expected porosity distribution as a fractal, using two different pore sizes dependent on the phase-holder. Experimental results and predictions of the Young’s modulus showed a decrease with increasing porosity. The values of E were 37.1 and 9.3 GPa for porosities of 30% and 70%, respectively. These results indicate the fractal behavior of the porosity of experimental metallic foams and demonstrate the effectiveness of the proposed fractal model in predicting the mechanical properties of these materials, which is an important tool in their design and manufacturing [23].

The application of the fractal approach also allows for the consideration of the influence of various additives [24] and non-metallic inclusions on mechanical properties [25], establishing a relationship between the fractal (fractional) dimension of porous material structures and their properties [26], and so on.

2 Materials and Methods

The material composition for the glass-basalt composite tubes was as follows (in %): 70% by weight is glass roving 1200 (grade E), and 30% is the binder (hardener brand IZOMTGFA and accelerator type Alcofen), with the Alcofen content up to 1%. The mass ratio of resin to IZOMTGFA in the binder was 100 to 80.

The microstructure and external appearance of the glass-basalt composite tubes are shown in Fig. 1.

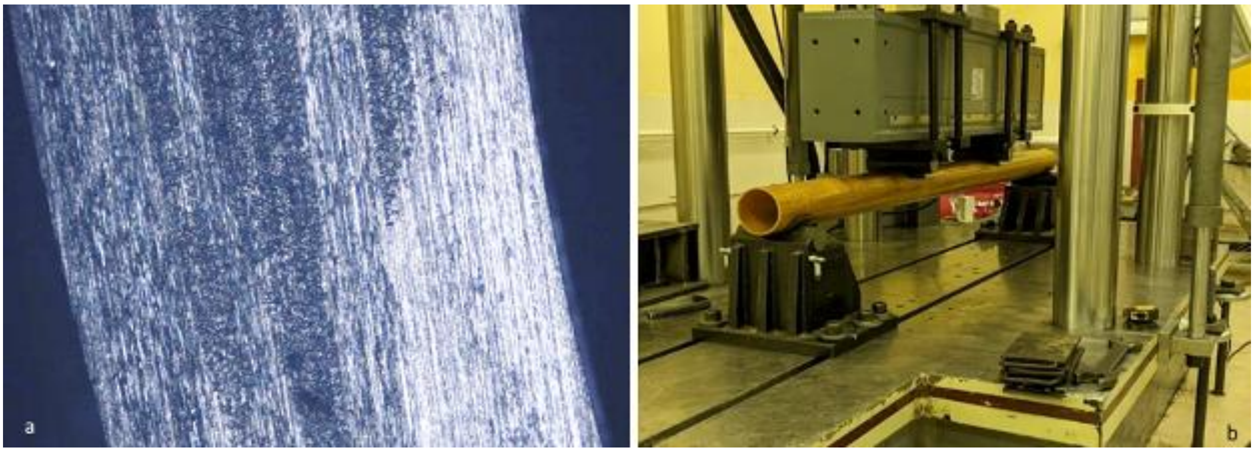

Fig. 1. – a - Microstructure and b - appearance of glass-basalt-plastic composite pipes during strength tests

Analysis of the investigated tubes indicates that they have a heterogeneous structure, for which the multifractal approach is appropriate.

3 Results and Discussion

It is well known that basalt fiber is a product derived from natural materials by melting and drawing them into fibers. Its primary purpose is to provide volume-dispersed reinforcement for various materials, including pipes. The chaotic arrangement of fibers within the material dictates its fractal nature.

Fractal dimensions D(q) of the heterogeneity indicators for the matrix fibers and the epoxy component of glass-basalt composite pipes were evaluated using a multifractal approach [27]. This approach is based on generating a measure that describes the partitioning of space, encompassing the studied element (the measure carrier), into Euclidean shapes such as squares, cubes, or circles, with a cell size ε. A software tool was developed with an algorithm based on the relationships (1)-(3) and consisting of five stages.

The spectrum of statistical dimensions D(q) was computed using the Renyi formula (3) [27]:

$$D(q) = \frac{1}{q-1} \cdot \lim_{\delta \to \infty} \frac{\ln \sum_{i=1}^{N} p_i^q}{\ln \delta}, \tag{3}$$

where $\sum_{i=1}^{N} p_i^q$ the exponent represents the statistical sum of probabilities p_i, which describe the probability of a point from the studied element (pixel) falling into the i - tn square cell δ.

The spectrum of fractal dimensions $f(a)$, which characterizes the distribution, was calculated based on Legendre transforms (5) for the function $\tau(q)$ [28]:

$$\begin{cases} \alpha = \dfrac{d\tau}{dq}, \\ f(\alpha) = q\alpha - \tau(q) \end{cases}. \tag{4}$$

The samples of the glass-basalt composite pipes were analyzed (Fig. 2), and their Young’s modulus was computed.

Fig. 3 shows the graphs of the distribution of dimensions $D(q)$ and heterogeneity indicators $f(\alpha)$ for the microstructure of hybrid glass-basalt composite pipes.

Fig. 4 compares the fractal dimension values and Young’s modulus indicators for the tested pipes. The experimental data points were approximated using linear models.

The accuracy of the approximation was described by the correlation coefficients R^2 of the equations, which were: for fractal dimension D_0 (with the exponent $q = 0$) 0.95, for D_1 0.92, for D_2 0.90, for D_{-200} 0.82, and for D_{200} 0.94. The R^2 values for the matrix fibers model were below 0.68 compared to other models. However, the dimension D_{200} characterizes the dimension of the matrix fibers and can be used for more accurate predictions of Young's modulus. The dimension of the dark elements of the structure (the epoxy component) corresponds to the dimension D_{-200}.

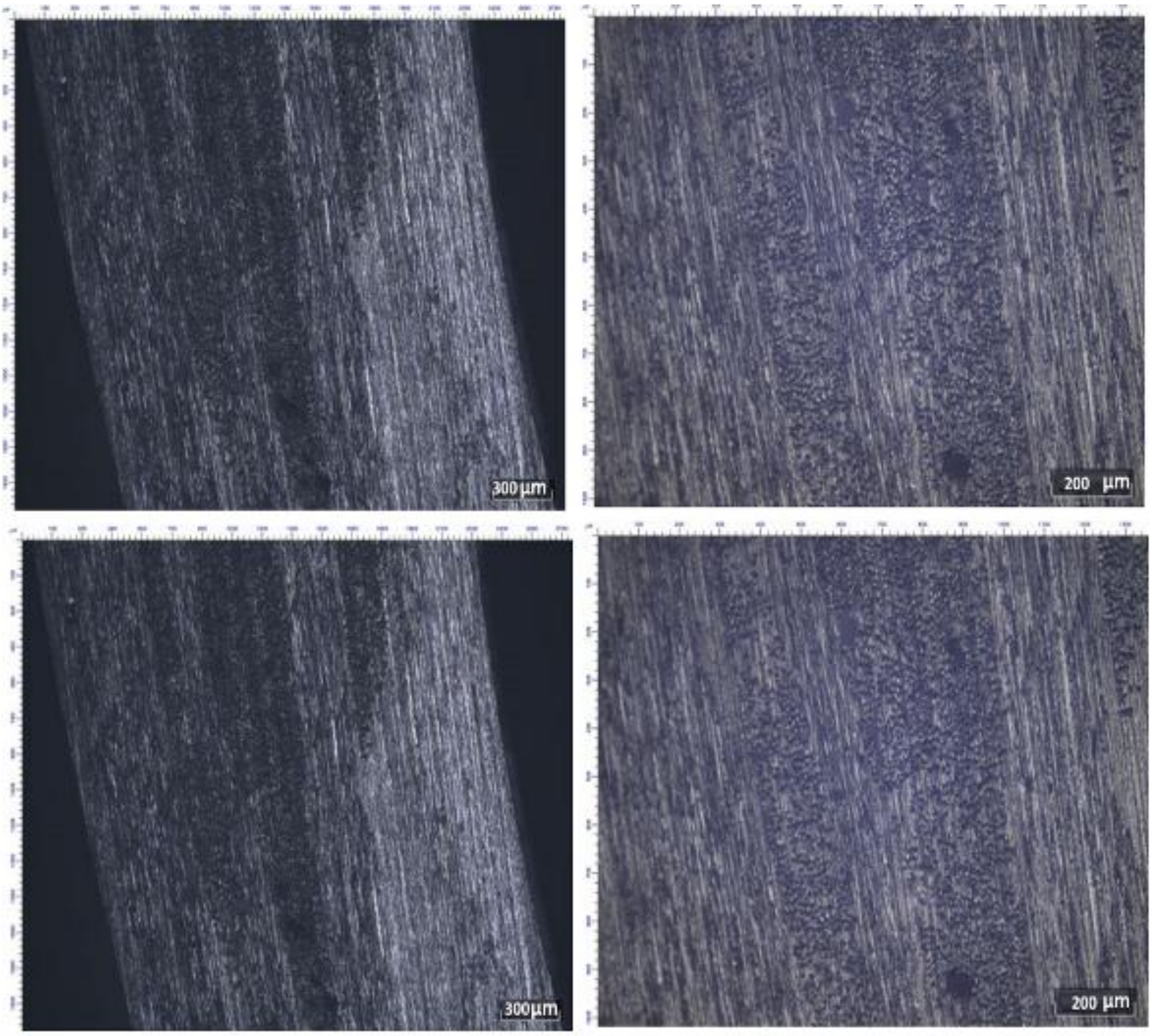

Fig. 2. – Microstructure of glass-basalt-plastic composite pipes: a – Fractal dimension of fibers for sample 1 - 1.72; b – for sample 2 - 1.86; c – for sample 3 - 1.88; d – for sample 4 - 1.94

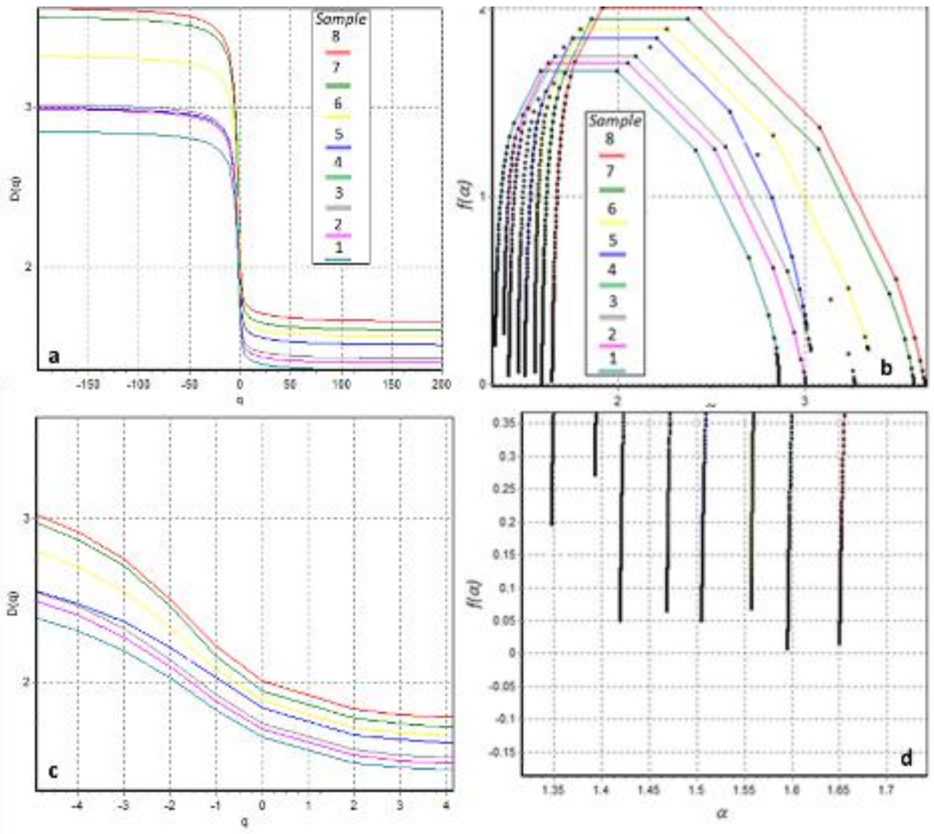

Fig. 3. – Spectrum of fractal dimensions $D(q)$ (a) and singularities $f(\alpha)$ (b), calculated for matrix fibers and enlarged fragments of these graphs

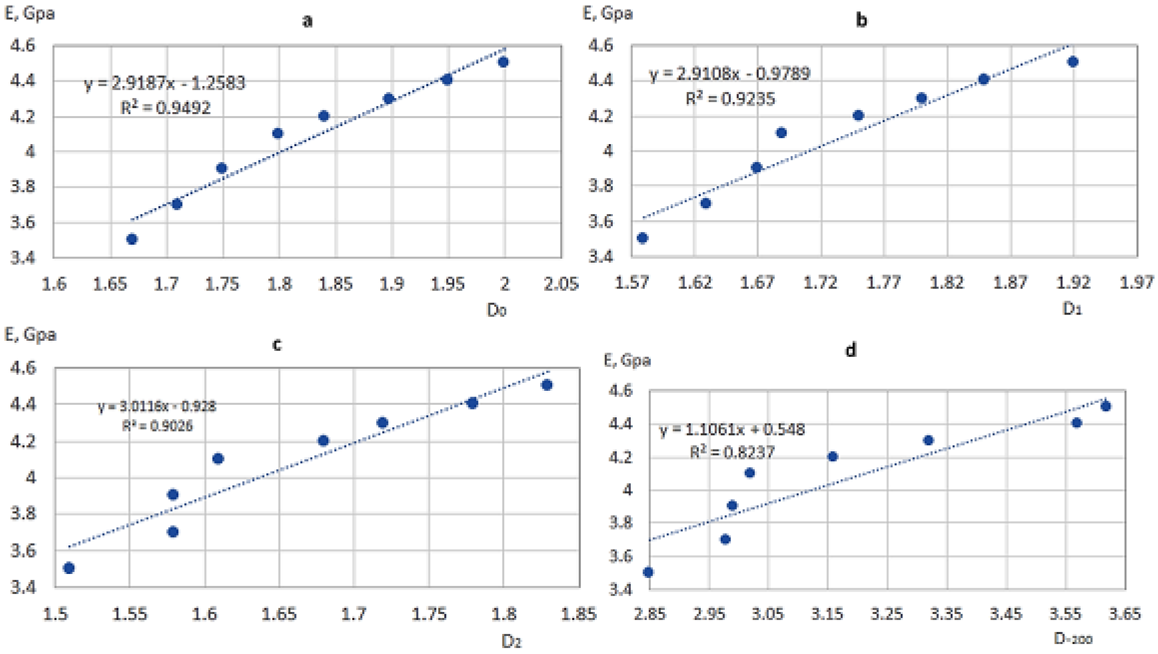

Fig. 4. – Relationships between fractal dimensions and heterogeneity indices and Young's modulus for glass-basalt-plastic composite pipes

The experimental results shown in Fig. 4 indicate a one-to-one correspondence between the fractal dimension indicators of the structural elements of composite pipes, heterogeneity, and Young's modulus. This allows for the implementation of real-time forecasting of the elastic modulus based on microstructure analysis.

4 Conclusions

1. This study presents an approach for evaluating the Young's modulus of glass-basalt composite pipes by applying multi-fractal analysis to the fiber matrix.
2. It was found that the strongest correlation between the modulus of elasticity and the fractal dimension was observed for the light inclusion dimension (fiber matrix) D_{200} ($R^2 = 0.94$) and the Hausdorff dimension D_0 ($R^2 = 0.95$), which can be attributed to the fractal nature of their structure.
3. The physical-mechanical properties of the composite pipes were also found to exhibit relatively high values, with compressive strength ranging from 2.6 to 3.6 MPa and tensile strength between 3.4 and 4.6 MPa.
4. The results suggest that the fractal dimension of the fiber matrix in glass-basalt composite pipes can serve as an indicator of structural changes, which in turn affect the material's Young’s modulus.

5 Fundings

This research was supported by a grant from the Russian Science Foundation No. 24-19-00691, rscf.ru/project/24-19-00691/.

References

1. Miodownik, M. Stuff matters: exploring the marvelous materials that shape our man-made world. Boston: Houghton Mifflin Harcourt, 2014.
2. Gordon, J.E. The new science of strong materials: or why you don't fall through the floor. Princeton: Princeton University Press, 2006. 58 p.

3. Vafaeva K.M., Zegait R. Carbon nanotubes: revolutionizing construction materials for a sustainable future: A review // Research on Engineering Structures and Materials. Elsevier Ltd, 2023. Vol. 10, No. 2, pp. 559–621. doi 10.17515/resm2023.42ma0818rv.

4. Vafaeva K.M., Duklan N., Mohan C., Kumar Y., Ledalla S. Comparative analysis of glass-basalt-plastic materials for construction in Arctic conditions // BIO Web of Conferences. EDP Sciences, 2024. Vol. 86, p. 01112. doi 10.1051/bioconf/20248601112.

5. Vafaeva K.M., Dhyani M., Acharya P., Parik K., Ledalla S. Glass-basalt-plastic materials for construction in temperate and Arctic climatic regions // BIO Web of Conferences. EDP Sciences, 2024. Vol. 86, p. 01111. doi 10.1051/bioconf/20248601111.

6. Vafaeva K.M., Chhetri A., Sudan P., Mishra M., Babu B.S., Mongal B.N. Polymer matrix nanocomposites for sustainable packaging: A green approach // E3S Web of Conferences. EDP Sciences, 2024. Vol. 537, p. 08001. doi 10.1051/e3sconf/202453708001.

7. Alaimo G., Valenza A., Enea D., Fiore V. The durability of basalt fibres reinforced polymer (BFRP) panels for cladding // Materials and Structures. 2016. Vol. 49, No. 6, pp. 2053–2064.

8. Chikhradze N.M., Japaridze L.A., Abashidze G.S. Properties of basalt plastics and of composites reinforced by hybrid fibers in operating conditions // Composites and their applications, 2012. p. 243-268.

9. The Rational mechanics of Flexible or Elastic Bodies, 1638–1788: Introduction to Leonhardi Euleri Opera Omnia, vol. X and XI, Seriei Secundae. Zurich: Orell Fussli, 1788.

10. Jastrzebski D. Nature and Properties of Engineering Materials. Wiley International Edition. New York: John Wiley & Sons, Inc., 1959.

11. Gorodtsov V.A., Lisovenko D.S. Extreme values of Young's modulus and Poisson's ratio of hexagonal crystals // Mechanics of Materials. 2019. Vol. 134, pp. 1–8. doi 10.1016/j.mechmat.2019.03.017.

12. Rahemi R., Dongyang L.I. Variation in electron work function with temperature and its effect on the Young's modulus of metals // Scripta Materialia. 2015. Vol. 99, pp. 41–44.

13. Zeng Q., Luo M., Pang X., Li L., Li K. Surface fractal dimension: an indicator to characterize the microstructure of cement-based porous materials // Applied Surface Science. 2013. Vol. 282, pp. 302-307.

14. Mahamud M., López Ó., Pis J.J., Pajares J.A. Textural characterization of coals using fractal analysis // Fuel Process. Technol. 2003. Vol. 81, pp. 127–142.

15. Duan Q., An J., Mao H., Liang D., Li H., Wang S., Huang C. Review about the application of fractal theory in the research of packaging materials // Materials. 2021. Vol. 14, p. 860. doi 10.3390/ma14040860.

16. Zeng Q., Luo M., Pang X., Li L., Li K. Surface fractal dimension: an indicator to characterize the microstructure of cement-based porous materials // Applied Surface Science. 2013. Vol. 282, pp. 302-307.

17. Dhand V., Mittal G., Rhee K.Y., Park S.-J., Hui D. A short review on basalt fiber reinforced polymer composites // Composites Part B: Engineering. 2015. Vol. 73, pp. 166-180.

18. Mahamud M., López Ó., Pis J.J., Pajares J.A. Textural characterization of coals using fractal analysis // Fuel Process. Technol. 2003. Vol. 81, pp. 127–142.

19. Volchuk V.M., Kotov M.A. Fractal express methods evaluation of a breaking stress of concrete // J. Phys.: Conf. Ser. 2021. Vol. 1926, p. 012023. doi 10.1088/1742-6596/1926/1/012023.

20. Kroviakov S., Volchuk V., Zavoloka M. Fractal model of the influence of expanded clay concrete macrostructure on its strength // KEM. 2020. Vol. 864, pp. 43-52. doi 10.4028/www.scientific.net/KEM.864.43.

21. Fang Q., Li B., Yin J., Yuan X. Microstructural and microanalytical study on concrete exposed to the sulfate environment // IOP Conference Series: Materials Science and Engineering. 2017. Vol. 269, p. 012070.

22. Volchuk V., Ivantsov S., Tiutieriev I., Fortyhin A. Search for the evaluation of ‘strength-plasticity’ relation in constructional steel // KEM. 2020. Vol. 864, pp. 211-216. doi 10.4028/www.scientific.net/KEM.864.211.

23. Pérez L., Lascano S., Aguilar C., Domancic D., Alfonso I. Simplified fractal FEA model for the estimation of the Young’s modulus of Ti foams obtained by powder metallurgy // Materials & Design. 2015. Vol. 83, pp. 276-283. doi 10.1016/j.matdes.2015.06.038.

24. Gao Y., Jiang J., De Schutter G., Ye G., Sun W. Fractal and multifractal analysis on pore structure in cement paste // Constr. Build. Mater. 2014. Vol. 69, pp. 253-261.

25. Volchuk V.M., Uzlov O.V., Puchikov O.V., Ivantsov S.V. Fractals theory application for evaluation of influence of non-metallic inclusions on mechanical properties of S355J2 steel // IOP Conference Series: Materials Science and Engineering. 2021. Vol. 1021, p. 012053.

26. Raoufi D., Fallah H.R., Kiasatpour A., Rozatian A.S.H. Multifractal analysis of ITO thin films prepared by electron beam deposition method // Applied Surface Science. 2008. Vol. 254(7), pp. 2168-2173.

27. Rényi A. Probability Theory. Amsterdam: North-Holland, 1970, p. 666.

28. Bozhokin S.V., Parshin D.A. Fractals and multifractals. Izhevsk: Regular and Chaotic Dynamics, 2001, p. 128.

Дата поступления: 13.10.2024

Дата публикации: 27.11.2024